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November 10, 2012 (Second Draft)

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Does arbitrage trading based on a cointegrating relation work? The efficient market hypothesis insists that there exists no successful trading. In order to avoid problems with data snooping, we choose 5 industrial average indices and a market index to make a cointegration analysis between 6 variables. Many cointegrating relations are found in 3 countries; Japan, the UK, and the USA. Using the facts, we simulate arbitrage trading between a market index and an industrial averages portfolio for each country. The calculation of this simulation shows that the arbitrage is successful. The average returns on the trading are monthly 0.3% to 0.5%.

1 Introduction

The efficient market hypothesis insists that there exists no successful trading strategy. For these decades there has been considerable literature to try to manifest statistical characteristics of stock prices. Various studies about anomalies are outstanding. Another is contrarian/momentum strategies, by which trade using past stock prices makes a positive return on average. This paper shows that an investment strategy that is quite different from the contrarian/momentum brings in a corresponding return.

An investment strategy in this paper is arbitrage trading based on a cointegrating relation. Arbitrage is the trade in which we take advantage of temporary deviation between two prices. When we find the deviation, the relatively cheap is bought and the relatively expensive is sold. The arbitrage supposes that the deviation is temporary and that it will be corrected sooner or later. It is no wonder that we achieve a gain if the deviation gets smaller after the trading. We can make a profit by ending the arbitrage when the deviation vanishes by convergence of the two prices.

The cointegration is a characteristic of time-series data in econometrics. This is embodied in our discussion as follows. The value of a portfolio at time t, W_t, which consists of n stocks, is denoted as

$$W_t = \sum_{i=1}^{n} a_i P_{i,t}.$$

A position of the ith stock is reflected into the value of a_i. $P_{i,t}$ is the stock price of i at time t. After the positions are formed, W_t varies due to a change in $P_{i,t}$. Even if W_t deviates from its initial value, W_t might be returned to it for a short period. When this reversion sometimes arises during a period, W_t can satisfy stationarity. It is highly probable that stock prices follow a unit root process rather than a stationary one. In the case of a unit root process, the deviation from its initial value continues for a long time, and the reversion to it rarely happens. However, if some prices which follow a unit root process are cointegrated, their linear combination, W_t, is stationary. A long or short position, a_i, is calculated from an estimate of a cointegrating vector.

If we find stocks of which a cointegrated system consists, we can do arbitrage trading with it. For simple explanation, we assume that W_t is constructed from two prices, $P_{M,t}$ and $P_{W,t}$, where $W_t = P_{M,t} - P_{W,t}$, and that the initial value of W_t is zero. Since stationarity makes negative W_t up to or positive W_t down to zero for a short period, W_t will revert to its initial value. Under movement of these prices, we can make a profit by buying $P_{M,t}$ and selling $P_{W,t}$ when W_t is negative. On the other hand, selling $P_{M,t}$ and buying $P_{W,t}$ profits when W_t is positive. Negative W_t means that $P_{M,t}$ is cheap relative to $P_{W,t}$, and $P_{W,t}$ is expensive relative to $P_{M,t}$. The cheap is bought and the expensive is sold. This trading is arbitrage, which supposes that these prices are convergent in the future through the reversion of W_t. If we construct a portfolio using a cointegrated system, this statistical characteristic describes that two prices are convergent with high probability, and the trading can be regarded as arbitrage expecting a profit through two prices' movement. We call this trading as arbitrage based on cointegration.

In this paper we investigate whether arbitrage trading based on cointegration is feasible. Returns are computed when we simulate executing the arbitrage. In order to avoid a data-snooping problem with selection of individual stocks, we choose 5 industrial average indices and a market index and analyze cointegration of 6 variables. [*1] These 6 variables are assumed to be ingredients of the arbitrage. We use 3 countries' stock market data; Japan, the UK, and the USA. Arbitrage trading in each country is studied in this paper. In these 3 countries, many cointegrating relations are found. Using the facts, we simulate arbitrage trading between industrial average indices and a market index. The result of this simulation shows that the arbitrage is successful. The monthly average returns on the trading are 0.3% to 0.5%. These 3 countries produce similar results.

There are many studies about the investment strategy that is contrarian/momentum using the USA equity data. A stock which has good performance is a winner, and a low return stock is a loser. Contrarian which sells winners and buys losers at the same time can make a profit on average: DeBondt-Thaler(1985), Lo-MacKinlay(1990), and Jegadeesh-Titman(1995). On the contrary, the strategy which sells losers and buys winners is mo-

[*1] We use the average of stock prices that belong to the same industry. In this paper it is denoted as an industrial average index.

mentum. A lot of papers find that arbitrage based on the momentum brings a positive return: Jegadeesh-Titman(1993), Fama-French(1996), Moskowitz-Grinblatt(1999), Grundy-Martin(2001), Jegadeesh-Titman(2001), Chen-Hong(2002), and Lewellen(2002).

Broadly speaking, above articles show that a monthly return on the strategy is 0.5% on its mean and 2% on its standard deviation. We should take care of how to calculate the return. Since the net position of an arbitrage portfolio is zero, it is difficult to define its return as the ratio of the amount of profits to the net position. Almost all articles calculate a return from the money a portfolio yields per a unit of its long position. In other words, divide the money an arbitrage portfolio earns with the amount of its buying position, and adjust it with the length of a period. This paper employs this method when calculating a return on our arbitrage trading.

This paper tests whether the arbitrage trading anticipates a profit. Section 2 shows how to deal with the arbitrage which concretely uses a cointegrating relation. Section 3 explains data when running simulation and summarizes results of cointegration analyses. Section 4 simulates arbitrage trading for every combination which we have found cointegrated. In Section 5, robustness is tested for results of Section 4. We confirm how results change when some assumptions of the simulation are altered. Section 6 concludes this paper.

2 Arbitrage Trading Based on a Cointegrating Relation

This section explains arbitrage trading based on a cointegrating relation. Although the arbitrage can be executed in various ways, we describe hereinafter the method by which our simulation implements the arbitrage.

In order to avoid problems with snooping data, we use a market index and industrial average indices instead of individual stock prices. These days contracts for difference, CFD, on many kinds of prices are becoming available around the world. It is not impossible to trade market and industrial average indices through CFD. The purpose of using the indices in this paper is to skirt how to choose properly among many stocks. It seems more practical to apply the method to individual stock investment.

This paper focuses on the case where we combine 5 industries. Their stock price average indices and a market index constitute 6 variables on which cointegration is tested. If they are cointegrated, their linear combination has stationarity. It means that when the linear combination of the 5 industrial averages is larger than the market index, then they can be inverted shortly afterwards. They can be reversed one after the other for a short period. Arbitrage can be executed with this characteristic.

It is a cointegrating vector which shows us how to construct a linear combination of 5 industrial average and market indices. Since a cointegrating vector is not unique, a coefficient on a market index is normalized to 1 in this paper. $P_{M,t}$ is a stock market index at time t, and $P_{W,t}$ is a linear combination of 5 industrial average indices. From a sample of monthly data on December 1970 through December 1999, an example where there exists a cointegrating relation is the combination of TOPIX and Japanese 5 industries of Chemicals, Forestry/Paper, Industrial Engineering, Leisure Goods, and Media. An estimated cointegrating vector for these industries shows that $W_t = P_{M,t} - P_{W,t}$ is stationary when $P_{W,t}$ is defined as

$$P_{W,t} = 2.517 \times P_{1,t} + 1.037 \times P_{2,t} - 0.492 \times P_{3,t} + 0.538 \times P_{4,t} + 0.198 \times P_{5,t},$$

where $P_{i,t}$ denotes an industrial average index for $i = 1, \cdots, 5$. $P_{1,t}$ is Chemicals, $P_{2,t}$ Forestry/Paper, $P_{3,t}$ Industrial Engineering, $P_{4,t}$ Leisure Goods, and $P_{5,t}$ Media.

If we take long or short positions for these 5 industrial average indices, $P_{W,t}$ represents the value of this industry portfolio. Substituting data on December 1999 into $P_{i,t}$ $(i = 1, \cdots, 5)$ of the above equation, $P_{W,t}$ is 1467.89. Since this is smaller than TOPIX of 1722.20, we set an industry portfolio long and TOPIX short.

We assume that an overall market and industrial averages are bought or sold for prices that are equal to values of their indices. When we purchase the industry portfolio for 1 million dollars, the portfolio is purchased with 681.2 units($= 1mil./1467.89$). To construct this portfolio, Chemicals are bought with 1714.6 units($= 681.2 \times 2.517$), Forestry/Paper with 706.4 units($= 681.2 \times 1.037$), Leisure Goods with 366.5 units($= 681.2 \times 0.538$), Media with 134.9 units($= 681.2 \times 0.198$), and Industrial Engineering is sold with 335.2 units($= 681.2 \times 0.492$). Since the industry portfolio is bought with 1 million dollars in the value of index 1467.89, TOPIX is sold with 681.2 units, which amount to 1,173,249 dollars($= 1mil. \times 1722.20/1467.89$).

Arbitrage begins by selling and buying the amounts calculated above. TOPIX and the value of the industry portfolio vary due to changes in prices subsequently, as shown in Table 1. On January and February 2000, TOPIX is still larger than the industry portfolio. Since they get opposite on March 2000, these prices converge and the arbitrage trading is over. Table 2 shows a profit of the trading, which comes from clearing these positions. Negative numbers of the column of “TOPIX” represent TOPIX short, and positive ones of “I.P.” mean buying the industry portfolio. What the trading has brought about is calculated in “Profit” on the table. Since TOPIX declines a little and the value of the industry portfolio rises during the period, we can gain profits from both the positions. The rightmost column “Asset” is the sum of investor’s net worth and the profit. The investor is assumed to have his or her own money of 1 million dollars at an initial date. His/Her net worth has increased to 1,173,336 dollars in three months. The average return on his/her initial net worth is 5.47% per month.

Table. 1 Series of Indices

t	$P_{M,t}$	$P_{W,t}$	$P_{1,t}$	$P_{2,t}$	$P_{3,t}$	$P_{4,t}$	$P_{5,t}$
1999/12	1722.20	1467.89	301.28	268.39	225.71	566.46	1197.46
2000/01	1707.96	1627.35	352.94	308.30	239.76	524.14	1286.66
2000/02	1718.94	1649.02	346.61	253.08	224.83	579.14	1578.76
2000/03	1705.94	1706.07	366.00	322.61	233.42	548.30	1361.70

$P_{M,t}$ is TOPIX, and $P_{i,t}$ $(i = 1, \cdots, 5)$ are industrial average indices. $P_{1,t}$ is Chemicals, $P_{2,t}$ Forestry/Paper, $P_{3,t}$ Industrial Engineering, $P_{4,t}$ Leisure Goods, and $P_{5,t}$ Media. Substituting data into $P_{i,t}$ $(i = 1, \cdots, 5)$, $P_{W,t}$ is the value of this industry portfolio.

Table. 2 Profits on the Arbitrage Trading

	TOPIX	I.P.	Profit	Asset
1999/12	-11732.5	10000.0		10000.0
2000/01	-11635.5	11086.3	1183.3	11183.3
2000/02	-11710.3	11233.9	1256.2	11256.2
2000/03	-11621.7	11622.6	1733.4	11733.4

Table 2 shows this arbitrage trading. Negative numbers of "TOPIX" represent dollar values of the TOPIX short position, and positive ones of "I.P." mean dollar values of buying the industry portfolio. What trading has brought about is calculated in "Profit." The sum of investor's net worth and the profit is "Asset" on the table.

If TOPIX is smaller than the value of the industry portfolio, then the arbitrage is composed of buying TOPIX and selling the industry portfolio. Since we assume that the net worth is used into a purchase side, TOPIX is bought with investor's money. Furthermore we assume that proceeds from selling short are left cash until the arbitrage positions are cleared.

The arbitrage trading we described above was able to terminate its positions 3 months later because prices converged as we had expected. If prices do not converge after the arbitrage started, what shall we do? When we simulate trading in this paper, we assume that an investor holds the positions for fixed months and that he or she closes the position to realize a profit/loss just after the months have passed.

3 Data and Cointegration Analyses

In order to simulate arbitrage trading between a market index and an industry portfolio, we obtain data on stock markets of three countries; Japan, the UK, and the USA. The market index, which investors turn their attention to, should reflect overall movement of a market. As for the market indices, we employ TOPIX in Japan, FTSE ALL-SHARE index in the UK, and S&P500 in the USA. On the other side, data of industrial average indices for the three countries are available in *Datastream* by THOMSON FINANCIAL. *Datastream* provides us with industrial average indices that are denominated as a DS industry price index. They are recalculated by THOMSON FINANCIAL from January 1973. In this paper samples begin with December 1979. The numbers of industries on December 1979 are 30 for Japan, 32 for the UK, and 38 for the USA. We depend on these industries when simulating the arbitrage trading.

In a software package the maximum number of variables is 6 in the Engle-Granger test of a cointegration analysis. We examine a cointegrating relation between 6 variables; an overall stock market index and 5 industrial average indices. This paper looks through 19 samples which consist of monthly data on the indices. These samples begin with December 1979 in common and end every six months from December 1999 to December 2008. In other words, there are 19 samples that are made up of the indices at the end of months from December 1979 to December 1999, from December 1979 to June 2000, $\cdots$, and from December 1979 to December 2008. As for these 19 samples, we investigate a cointegration analysis of 6 variables where we choose 5 from 30 industries for Japan, 5 from 32 industries for the UK, and 5 from 38 industries for the USA.

The indices that are to be cointegrated must be a unit root process. Although results are omitted, there are a few industrial averages that are not regarded as a unit root process. We test all the indices through ADF in three cases of *no constant*, *constant and no trend*, and *constant and trend*. The indices that the three tests prove to have a unit root are adopted in our cointegration analysis. The numbers of the industrial averages with a unit root for the samples are summarised in “Ind” of Table 3. Market indices of Japan, the UK, and the USA pass the tests.

Over a sample, for example, there are 36 industrial average indices that have a unit root. Combinations of 5 out of 36 industries are more than 370 thousands. We confirm if they are cointegrated for every combination. For each country and sample, “Coint” of Table 3 shows the number of combinations that have a cointegrating relation. A method depends on the Engle-Granger test in the case of *constant and no trend*.

From the total of the 19 samples, more than 100 thousand combinations are found to be cointegrating in each country. These numbers are enough to simulate arbitrage trading based on cointegration. The UK has the least 150 thousand combinations, and the USA has more than 540 thousands. Japan has 220 thousands. Overlooking the 19 samples, cointegrating relations increase or decrease with each sample period. In Japan, samples of the first half periods find more cointegrating combinations than the latter ones. By contraries, the UK has much more in the latter half periods. But two samples after the Lehman shock have less combinations in the UK. The USA also has more cointegration in the latter half, which is less obvious than the UK.

Table. 3 The Number of Industries and Combinations That Have a Cointegrating Relation

country	Japan		United Kingdom		United States	
sample	Ind.	Coint	Ind.	Coint	Ind.	Coint
1999/12	30	17010	23	779	34	13916
2000/06	28	18693	22	1421	30	9805
2000/12	28	19577	23	2264	30	17541
2001/06	28	18415	24	3599	33	30084
2001/12	28	15930	23	2697	32	25861
2002/06	28	12787	25	3321	34	35521
2002/12	29	15174	27	7598	33	27863
2003/06	29	14553	27	7586	30	17217
2003/12	29	10863	23	4156	32	28647
2004/06	28	7785	24	5708	34	38773
2004/12	30	12525	28	13494	33	29584
2005/06	30	11605	28	14562	33	30531
2005/12	28	7198	28	13377	34	29396
2006/06	29	8856	28	14237	35	37873
2006/12	29	8408	28	12704	35	32490
2007/06	29	8380	30	20476	36	40790
2007/12	28	6187	28	15215	36	41082
2008/06	28	6162	24	6273	34	30129
2008/12	25	2935	26	9319	31	24589
Total		223043		158786		541692

This table summarizes the number of industries of which the average index follows a unit root process, and the number of combinations in which a cointegration test concludes that there exists one cointegrating relation. All the indices are tested through ADF in three cases of *no constant*, *constant and no trend*, and *constant and trend*. The indices that the three tests prove to have a unit root are adopted in our cointegration analysis. The numbers of the industrial averages with a unit root for the samples are summarised in "Ind" of Table 3. "Coint" shows the number of combinations that have a cointegrating relation. A method depends on the Engle-Granger test in the case of *constant and no trend*. "1999/12" means that estimation is obtained from the sample period on December 1979 through December 1999. For each sample until "2008/12" the unit root test and the cointegration analysis are executed.

4 Simulation of Arbitrage Trading

We have simulated arbitrage trading using the combinations of industries whose indices are cointegrated. Simulation results are summarized on Table 4. Constructing a portfolio with weights derived from an estimate of a cointegrating vector, we calculate the profit/loss of the arbitrage trading which uses the portfolio for at most 6 months. If the value of the portfolio and a market index are reversed during the 6 months, the arbitrage is terminated at that time and its return is calculated from the value of the positions. Table 4 aggregates the returns for countries and periods.

Table. 4 Arbitrage Returns

			monthly return			%		
Country	Period	Num.	Mean	S.D.	t	Plus	Grp.0	Grp.1
JP	ALL	223043	0.00374	0.0214	82.5	50.5	77.4	22.6
	[1]	143002	0.00519	0.0227	86.6	51.9	76.6	23.4
	[2]	80041	0.00113	0.0186	17.2	47.9	78.8	21.2
UK	ALL	158786	0.00497	0.0180	109.8	58.8	62.4	37.6
	[1]	33421	0.00619	0.0179	63.1	58.0	68.7	31.3
	[2]	125365	0.00465	0.0181	91.1	59.0	60.7	39.3
USA	ALL	541692	0.00484	0.0219	162.8	55.1	65.2	34.8
	[1]	206455	0.00522	0.0232	102.1	55.7	67.7	32.3
	[2]	335237	0.00461	0.0210	126.9	54.7	63.6	36.4

This table shows the means and standard deviations of arbitrage portfolio returns when using all cointegrating vectors. Monthly returns are decimal. ALL of “Period” means that all 19 samples are aggregated. [1] includes 9 samples which end on 1999/12, 2000/06, $\cdots$, or 2003/12. [2] includes other 10 samples which end on 2004/06, 2003/12, $\cdots$, or 2008/12. “Num.” is the number of combinations on which a cointegrated relation exists. “Mean” denotes the mean for the combinations, “S.D.” the standard deviation, and “t” the t-value to test a zero mean. “Plus” is the percentage of positive returns over the sample. “Grp.0” is the percentage of cases where the arbitrage is not over during the maximum investment period that is 6 months. “Grp.1”is the percentage of cases where the arbitrage is terminated because the values of a stock market index and an industrial portfolio have been interchanged.

In Japan there are 223,043 combinations cointegrating. When simulating the arbitrage trading for all the combinations, the mean of monthly returns is 0.374%, and its standard deviation is 2.14%. These are made up of the 19 samples and denoted as ALL of “Period.” [1] and [2] of “Period” are results when the 19 samples are divided into the first and the second half. [1] is the first half, consisting of 9 samples in which each period ends on every 6 months from December 1999 to December 2003. The second half is [2], which gathers 10 samples from June 2004 to December 2008. Period [1] in Japan has good performance; its mean return is 0.519% and its standard deviation is 2.27%. The mean and the standard deviation decreases to 0.113% and 1.86% in period [2]. Mean returns are all significant from 0 in terms of t-values.

Table 4 also shows how many times the arbitrage trading has a positive return and how it has terminated. Simulation assumes that the maximum of a trading period is 6 months. Group 1 is the trades where trading ends less than 6 months with the reversal of the industrial portfolio value and the market index in between. In Group 0 the trading continues its positions for 6 months without the reversal and is forced to finish them just after the maximum period. The trading of the Group 1 gets a strictly positive return, but the return for Group 0 can be positive or negative. If the difference between the industrial portfolio value and the market index has diminished since the beginning of the trading, it ends with a positive return. “Plus” of Table 4 is the proportion of the positive return. “Grp.1” and “Grp.0” are the percentage of Group 1 and Group 0. In the case of Japan over all periods, half of the trading, 50.5%, make a profit, 22.6% get the reverse in less than 6 months. As well as the average returns, these proportions decrease in the latter half periods.

The UK and the USA have better outcomes than Japan over all periods. The UK has 0.497% on a return average and 1.8% on its standard deviation. The USA has 0.484% on an average and 2.19% on its standard deviation. The proportions of positive returns are 58.8% in the UK and 55.1% in the USA. The percentages of Group 1 are 37.6% in the UK and 34.8% in the USA. Though their returns decrease in the latter half periods like Japanese ones, it is not so much decline as Japan. The proportion of positive returns does not change for each period.

5 Robustness Tests

In order to test whether the results of Section 4 are robust, we examine how they will be varied when changing setups for simulating arbitrage trading. In the following, we investigate the effect of maximum investment periods, termination conditions, and initial differences.

5.1 Maximum Investment Period

In Section 4, the arbitrage trading will be terminated when two prices are convergent. Yet, in the case where they do not converge, the arbitrage trading continues for several months, which we call a maximum investment period. We have tried four maximum investment periods; 3 months, 6 months, 9 months, and 12 months. Results are shown on Table 5. The longer a maximum investment period is, the average returns on the arbitrage trading tend to be higher. It takes the highest average return at 12 months in the UK and the USA, and at 9 months in Japan.

The most interesting in changing the maximum investment period is that the increase in an average return does not always raise its standard deviation. In the longer period, the standard deviations decrease slightly. Moreover, the chance of price convergence improves with a longer investment period. It can be seen from Table 5 that the proportion of Group 1 increases for the three countries. However, we should be aware that it is difficult to know ex ante whether this investment strategy results in successful or not. It is just shown that the possibility of converging slightly increases with a longer investment period.

The success of this investment strategy depends on whether a difference between a stock market index and an industry portfolio value expands or not, and we cannot know it in ad-

Table. 5 The Effect of Maximum Investment Periods on Arbitrage Returns

	monthly return			%		
Length	Mean	S.D.	*t*	Plus	Grp.0	Grp.1
country: Japan						
3	0.00371	0.0234	74.9	53.5	85.3	14.7
6	0.00374	0.0214	82.5	50.5	77.4	22.6
9	0.00491	0.0202	114.6	53.9	72.9	27.1
12	0.00487	0.0201	114.5	51.4	69.4	30.6
country: United Kingdom						
3	0.00357	0.0200	71.2	54.9	74.3	25.7
6	0.00497	0.0180	109.8	58.8	62.4	37.6
9	0.00654	0.0168	155.0	64.6	53.9	46.1
12	0.00669	0.0168	158.6	66.1	48.0	52.0
country: United States						
3	0.00337	0.0241	103.1	52.6	75.3	24.7
6	0.00484	0.0219	162.8	55.1	65.2	34.8
9	0.00600	0.0205	215.1	56.6	59.4	40.6
12	0.00651	0.0201	238.4	58.2	54.8	45.2

This table shows the means and standard deviations of arbitrage portfolio returns when a maximum investment period is changed. Unless prices are convergent, an arbitrage portfolio are obliged to close its positions just after the period has passed. "Length" is the maximum investment period which is 3, 6, 9, or 12 months. Monthly returns are decimal. "Mean" denotes the mean for ALL, "S.D." the standard deviation, and "*t*" the t-value to test a zero mean. "Plus" is the percentage of positive returns. "Grp.0" is the percentage of cases where the arbitrage is not over during a maximum investment period. "Grp.1"is the percentage of cases where the arbitrage is terminated because the values of a stock market index and an industrial portfolio have been interchanged.

vance. Observing "Plus" of Table 5, the number of positive returns is larger than the one of negative returns. The longer the maximum investment period, the combinations where arbitrage ends with a positive return tend to increase. This is maximized at 12 months in the UK and the USA, and at 9 months in Japan.

5.2 Termination Conditions

As mentioned before, the arbitrage trading will be terminated when prices have converged or when the maximum investment period has passed. Here we add a condition of trading termination. The condition is that a difference between a stock market index and an industry portfolio value fluctuates beyond a margin from an initial date. For example, if the difference is 25% at the start and if the margin is 10%, the trading will be terminated when the difference decreases below 15% or increases above 35%. In the case of 15% margin, the trading is over when the difference decreases below 10% or increases above 40%. Table 6 shows simulation

Table. 6 The Effect of Termination Conditions on Arbitrage Returns

	monthly return			%			
Close	Mean	S.D.	*t*	Plus	Grp.0	Grp.1	Grp.2
country: Japan							
NO	0.00374	0.0214	82.5	50.5	77.4	22.6	0.0
15%	0.00327	0.0246	62.7	50.1	70.7	21.8	7.5
10%	0.00299	0.0280	50.3	49.6	59.9	20.4	19.7
country: United Kingdom							
NO	0.00497	0.0180	109.8	58.8	62.4	37.6	0.0
15%	0.00474	0.0189	99.9	58.7	58.5	37.5	4.0
10%	0.00425	0.0204	83.1	58.4	50.4	37.0	12.6
country: United States							
NO	0.00484	0.0219	162.8	55.1	65.2	34.8	0.0
15%	0.00443	0.0249	130.9	55.0	59.3	34.7	6.0
10%	0.00443	0.0286	113.9	55.0	50.1	34.3	15.6

This table shows arbitrage portfolio returns when a termination condition is set. Unless prices are convergent, or when the maximum investment period has passed, an arbitrage portfolio are obliged to close its positions just after a difference of the prices varies beyond a margin, compared with its initial value. The margin is 10% or 15%. For example, if the difference is 25% at the start, and if the margin is 10% of "Close", the trading will be terminated when the difference decreases below 15% or increases above 35%. The group of the trading that terminates by the condition is denoted as Group 2. "Grp.2" of Table 6 is the proportion of Group 2. NO of "close" denotes the results when the terminal condition is not added, which are reprinted from Table 4. Monthly returns are decimal. "Mean" denotes the mean for ALL, "S.D." the standard deviation, and "*t*" the t-value to test a zero mean. "Plus" is the percentage of positive returns. "Grp.0" is the percentage of cases where the arbitrage is not over during a maximum investment period. "Grp.1"is the percentage of cases where the arbitrage is terminated because the values of a stock market index and an industrial portfolio have been interchanged.

results when this termination condition is set. NO of "Close" indicates that the termination condition is not set, and these are the same as Tables 4.

The termination condition reduces the number of the trading that continues for the maximum periods. The proportions of Group 1 as well as Group 0 decrease in the case of the 15% margin and decrease more in the case of the 10% margin. The group of the trading that terminates by the condition is denoted as Group 2. "Grp.2" of Table 6 is the proportion of Group 2.

When adding the termination condition, returns on the arbitrage trading decline for all cases. Remarkable is that standard deviations increase while average returns decline. This condition shortens an investment period. As shown in Table 5, shortening investment period brings about higher risk and a lower return. These results suggest that performance of trading goes worse if we add to an inappropriate condition. Meanwhile, clearing positions due to price convergence or a maximum investment period is sufficient for simulating the arbitrage.

Table. 7 The Effect of Initial Differences on Arbitrage Returns

	monthly return			%
Quart.	Mean	S.D.	t	Plus
country: Japan				
Q1	0.00865	0.0208	98.3	63.2
Q2	0.00397	0.0223	42.0	47.3
Q3	0.00217	0.0234	21.9	44.5
Q4	0.00016	0.0179	2.2	47.0
country: United Kingdom				
Q1	0.00827	0.0170	96.9	72.0
Q2	0.00641	0.0184	69.6	58.2
Q3	0.00344	0.0184	37.2	51.9
Q4	0.00177	0.0176	20.0	53.0
country: United States				
Q1	0.00845	0.0190	163.7	68.0
Q2	0.00621	0.0206	110.7	55.1
Q3	0.00310	0.0211	53.9	49.6
Q4	0.00160	0.0256	23.0	47.7

This table shows arbitrage portfolio returns when initial differences of prices are divided into four groups. Q1 of "Quart." of Table 7 is the first quartile which gets the smallest differences. Q4 of "Quart." is the fourth quartile which has the largest. Monthly returns are decimal. "Mean" denotes the mean for a group, "S.D." the standard deviation, and "t" the t-value to test a zero mean. "Plus" is the percentage of positive returns.

5.3 Initial Differences

In this paper, arbitrage trading is started regardless of an initial difference between a stock market index and an industry portfolio value. However, this difference at a starting date is various from under 1% to several hundreds percent. According to Gatev-Goetzmann-Rouwenhorst(2006), a large difference at the start has brought a good outcome. So, by arbitrage trading in this paper, similar relations might be found between the initial difference and an arbitrage return. We investigate this on Table 7.

Table 7 assorts arbitrage trading into four groups by the initial difference. Q1 of "Quart." represents the first quartile which has the smallest difference, and Q4 of the column denotes the fourth which has the largest. If the above description were in the right, average returns should become larger from Q1 to Q4. Yet, we cannot find this tendency at all. Results on Table 7 are quite opposite to Gatev-Goetzmann-Rouwenhorst(2006). The smaller initial differences make the average returns higher.

Table 8 looks into the average return in the case of extreme small initial differences. "Ini-

Table. 8 Arbitrage Returns in the case of small initial differences

		monthly return			%
Initial	Num.	Mean	S.D.	*t*	Plus
country: Japan					
0.01	12892	0.00991	0.0190	59.1	73.7
0.02	25418	0.00986	0.0198	79.6	70.3
0.03	37872	0.00950	0.0203	91.0	67.1
country: United Kingdom					
0.01	21471	0.00856	0.0165	75.9	76.1
0.02	41288	0.00827	0.0171	98.5	71.7
0.03	59866	0.00784	0.0175	109.8	68.0
country: United States					
0.01	70249	0.00853	0.0184	122.7	71.6
0.02	137468	0.00843	0.0190	164.4	67.8
0.03	199855	0.00803	0.0195	184.0	64.6

This table looks into average returns in the case of small initial differences. “Initial” of the table denotes the initial difference; for example, 0.02 is below 2 percent. Monthly returns are decimal. “Num.” is the number of the trading for a group. “Mean” denotes its mean, “S.D.” the standard deviation, and “*t*” the t-value to test a zero mean. “Plus” is the percentage of positive returns.

tial” on the table denotes the initial difference whose value is below 1 percent, below 2 percent, and below 3 percent respectively. Table 8 shows the number of the trading, the mean and the standard deviation of returns, and the percentage of positive returns for each case. The three countries have the same features: The smaller gets the initial difference, the more positive is the trading return. Reflecting the fact, the average return increases and the standard deviation decreases. The arbitrage trading whose initial difference is below 1 percent ends with a positive return in 70%. In other words, these trades make a profit with 70% probability.

6 Conclusion

In this paper we investigate whether arbitrage trading based on cointegration is feasible. If some prices which follow a unit root process have a cointegrating relation, their linear combination is stationary. A stationary process might be returned to its initial value for a short period. This characteristic can make an arbitrage between prices which composes cointegration. In order to avoid a data-snooping problem with selection of individual stocks, we choose 5 industrial average indices and a market index and analyze cointegration of 6 variables. We use 3 countries’ stock market data; Japan, the UK, and the USA. Arbitrage trading in each country is studied. These 3 countries produce similar results. In these countries, many cointegrating relations are found. Using the facts, we simulate arbitrage trading between a market index and industrial averages. Returns are calculated when we simulate executing the arbitrage. The calculation of the simulation shows that the arbitrage is successful. The monthly average returns on the trading are 0.3% to 0.5%.

In terms of the average return the arbitrage trading based on cointegration can stand comparison with the contrarian/momentum strategy studied in the USA. Their standard deviations also seem alike. There might be criticism that the return is not corresponding to the risk in spite of troublesome procedure. Simulation brings results that half of trades make a loss. This paper shows that the performance improves when the arbitrage trading is limited to its small initial difference.

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