Final take-home exam Time Series Analysis

January 10, 2018

1. Solve all the theoretical homework given throughout the semester.

2. Show that the sum of an MA(1) process and a white noise process independent of the MA(1) process remains an MA(1) process. What happens if the white noise is possibly correlated with the MA(1) process?

3. A weakly-stationary time series is altered due to a partial loss of information. For each data-point, there is a probability $p > 0$ that the information is lost, and this happens independently from one point to another one. Accordingly, we create a new time series from the original one where we replace the values of the data points with loss of information by 0. Show that this new time series is also weakly-stationary, and express its autocorrelation function with respect to the original time series.

4. Using the appropriate package and functions, simulate 1000 points of a time series for the two models ARMA(1,1) (with values of parameter 0.2 for the intercept, 0.1 for the autoregressive part, 0.3 for the moving-average part and 1 for the variance of the noise, i.e. $X_t = c + \phi X_{t-1} + \theta\epsilon_{t-1} + \epsilon_t$ with $c = 0.2$, $\phi = 0.1$, $\theta = 0.3$, and $\mathbb{E}\epsilon_t^2 = 1$) and MA(3) (with 1 for all the five parameter values). On the basis of those simulated points, estimate the values of the parameters for both models. Do this for $M = 1000$ different paths. For each model, and each $i = 1, \cdots, M$, keep the estimated values of the different parameters. Report the mean and the standard deviation of the vectors of estimated values. Provide also the related histograms (only for the parameters of the ARMA(1,1)). Comment.

5. Fit MA(1), ARMA(1,1), MA(2) models to the stock prices available in EuStockMarkets dataset (just type EuStockMarkets in the R console), and report the estimated parameters and the AIC values. Which model would you choose based on this criterion ? Compute the forecast for the day after the last day included on the dataset. Based on the forecast, do you think there is any reason to choose one particular model? Now do the forecast on the 1000 last days of the sample, pretending that you don't observe the value on that day. Based on all the forecasting values and the observed values, do you think there is a reasonable way to choose one particular model now?